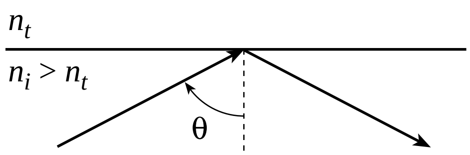
II. Light is a Ray (Geometrical Optics)

II.J. Ray Optics Description of a Waveguide — Trapping Light by TIR

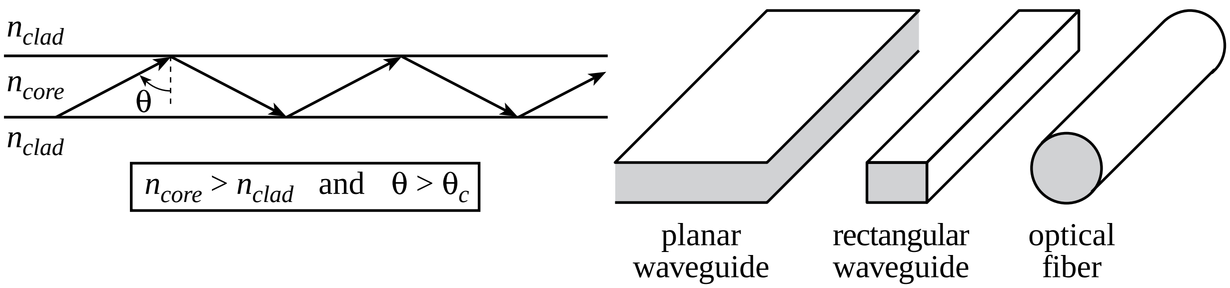
An <u>optical waveguide</u> is a structure that "guides" a light wave by constraining it to travel along a certain desired path. Usually the light is guided by Total Internal Reflection (TIR). Recall that TIR occurs when light is incident on a dielectric interface at an angle greater than the critical angle θ_c:



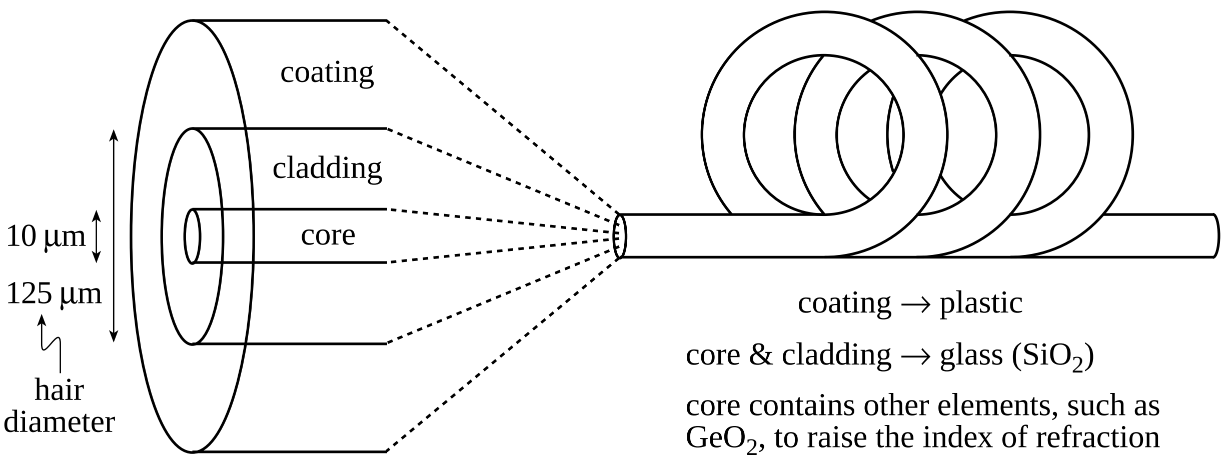
when $\theta > \theta_c = \textit{critical angle} = \arcsin(n_t / n_i)$

the light is totally reflected!

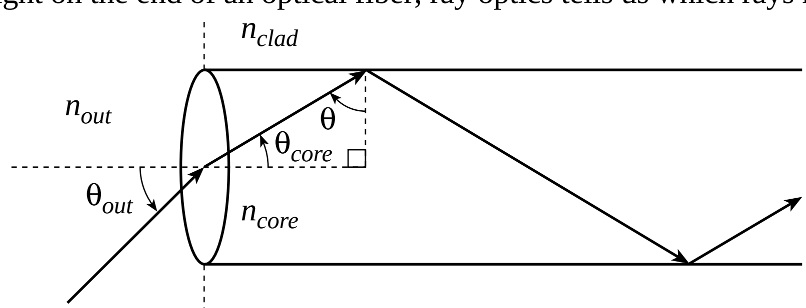
A waveguide traps light when more than one TIR interface (for <u>planar waveguides</u> and <u>rectangular waveguides</u>), or an interface that surrounds the guiding region (for <u>optical fibers</u>) is present.



The structure and typical dimensions of an optical fiber are shown up close in the diagram below.



When we shine light on the end of an optical fiber, ray optics tells us which rays may be guided.



We know that inside the fiber TIR occurs only for rays with $\theta > \theta_c$. We can work backwards to see what this condition tells us about the outside ray angle θ_{out}.

Snell's Law requires $n_{out} \sin\theta_{out} = n_{core} \sin\theta_{core}$,

but $\sin\theta_{core} = \cos\theta = \sqrt{1-\sin^2\theta}$ (where we use $\sin^2\theta + \cos^2\theta = 1$).

The largest ray angle incident on the end of the fiber that may be guided, which we denote θ_{out}^{max}, corresponds to the internal angle θ that is just equal to θ_c. Therefore

$$n_{out} \sin\theta_{out}^{max} = n_{core}\sqrt{1-\sin^2\theta_c} = n_{core}\sqrt{1-\frac{n_{clad}^2}{n_{core}^2}} = \sqrt{n_{core}^2 - n_{clad}^2}\ .$$

This result is often stated in terms of an important quantity in optics called the Numerical Aperture of an optical system:

Numerical Aperture (NA):	*the* sine *of the largest incident ray angle θ_{out}^{max} (measured with respect to the optical axis) that is successfully transmitted through an optical system, multiplied by the index of refraction of the medium just in front of the optical system, n_{out}, or* $NA \equiv n_{out} \sin\theta_{out}^{max}$.

Therefore, for an optical fiber we find

$$\boxed{NA = \sqrt{n_{core}^2 - n_{clad}^2}}\ .$$

In summary, if we know the indexes of refraction of the core and cladding of an optical fiber or waveguide, then we can immediately determine the *NA* of the waveguide, which tells us the largest cone of rays that may be successfully guided.

Notice that even though glass fibers are remarkably flexible (they truly are the optical analog of electrical wiring!), because the light is guided by TIR an optical fiber or waveguide may not be bent arbitrarily tightly without allowing some or all of the light to escape from the waveguide.