Research Projects, Warwick 2008

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2 Research

2.1 Motivation and Introduction

In our research we look at fundamental fluids from both computational and analytic (mathematical) perspectives, interleaving these wherever possible. Most of the flows obey are turbulent or the underlying Navier-Stokes equations have instabilities that could lead to turbulence. One of the underlying difficulties of turbulent flows is that the range of scales in an engineering/geophysical/astrophysical flow would require capabilities, primarily memory, that are far beyond any proposed computer. The problem can be reduced by considering: 1.) Steps in the formation of a turbulent flow; 2.) Simulate flows that are less turbulent (lower Reynolds number); 3.) Parameterise either the forcing or turbulent dissipation. All three are mentioned here.

Modern mathematics underlies these approaches, including whether singularities (infinities) form from smooth initial conditions in solutions of the Navier-Stokes equations in finite time has been chosen to be one of the 10 outstanding mathematical questions of our day. See: **www.claymath.org/millennium/**.

2.2 Methodology

Direct numerical simulations (DNS) form the core of this programme, with one of the objectives being improved parameterisations of large-eddy simulations (LES). In DNS, either viscosity is identically zero (Euler) or Newtonian (Navier-Stokes). In LES the turbulent transport and dissipation of kinetic energy are are parameterised by eddy-viscosity models. Most of the DNS uses traditional spectral methods. For the Euler equations the advantage is that energy is conserved exactly. It is hoped that LES will become the method of choice for industrial applications, therefore development is done with traditional finite-difference or finite-volume methods. We plan to use imported adaptive mesh methods, either spectral-element or finite-difference.

2.3 "Mathematical Analysis of the Euler Equations"

It was the understanding that the maximum of vorticity controls all pausible singularities [1] that led to the first numerical attempts at resolving the singularity questions in the Navier-Stokes equations and the inviscid Euler equations. Comparison with later bounds [Kerr05a] is the next step. Continued analysis of the underlying equations is therefore a necessary part of this numerical programme. Such considerations has led to new requirements for initalisation, new mathematical constraints, and new ideas for numerical analysis. Important recent advances are a new definition of the Lagrangian frame through the use of quaternions (first introduced by Hamilton) and a constraint on the types of time development. [2, 3]

2.4 "Numerical Simulation of the Euler Equations"

To follow the evolution of possible singularities requires the finest resolution possible. The objective is to accurately observe how properties such as local vorticity, circulation and energy do, or do not, collapse to singular surfaces as in Fig. 1. Whether a given simulation obeys all bounds provided by mathematical analysis and conserves all invariants is often controversial [4, 5, 6]. The first simulation to generally meet these criteria was [7]. We have recently begun a new series of calculations with resolution of up to $1024 \times 2048 \times 256$ points that consider the effects of different initialisation procedures and draw upon the mathematical insight gained since 1993 [8].Preliminary conclusions are that the 1993 data is better than thought, but many of the assumptions made in the analysis of that data were wrong. To draw further conclusions will require refined calculations; refined in the sense of more mesh points, taking the recently earned insight into initialisation to a higher level, and collaboration with people with adaptive mesh codes. This will require a multi-institutional effort led by Warwick. The first steps have been made with the group of R. Grauer in Bochum, Germany [9]and within the UK for forming a new UK Vortex and Superfluids HPC consortium for accessing national resources.

2.5 "Parameterisation of the Navier-Stokes Equations"

Improving parameterisations of the Navier-Stokes equations that can be used within large-eddy simulations requires understanding the dynamics at the smallest scales, in particular small-scale vortex dynamics and vortex reconnection as in Fig. 2. This is tied to the Euler singularity work because we believe that the Euler dynamics drives vortices together before reconnection. However, in this case more general flows need to be considered [10]. For that purpose we have acquired a large (2048^3), modern DNS of isotropic, decaying turbulence from Los Alamos National Laboratory (LANL) in New Mexico.

We have recently compared a class of nonlinear stress models with DNS, and then tested these within a traditional LES framework [11] The promise of nonlinear models is that they preserve many of the topological invariants of the full equations and there were favourable comparisons with the earlier idealised DNS [10] However, few improvements were found in comparison with older LES parameterisations. A major objective of M.A. Kopera, an EPSRC supported HPC student, will be to extend this type of analysis to more realistic wall-bounded flows.

Thus, we are now taking a step back and are considering another class of models [12] which makes more detailed predictions of fundamental turbulent statistics. The models use restricted singular dynamics to predict a particular alignment of second and third-order stress correlations, which is in fact observed. However, Fig. 3 shows that this is not associated with large vorticity, which is required mathematically [1]. This work is ongoing and will use the LANL data set. L. Chevillard has returned to Lyons from Johns Hopkins and will visit us shortly.

The terms in these models are closely related to the terms responsible for the generation of acoustic noise in turbulence, which ties this work to one of the major programmes in Engineering: Aerodynamics & Aeroacoustics of Co-flowing Jets, Grant Number. Engineering DTA student A.N. Ferguson is assigned to this project.

2.6 "Atmospheric Structure Functions"

With G King we are continuing to study structure functions in the atmospheric boundary layer. At mid-latitudes our third-order structure functions corroborate controversial results that the -5/3 spectral regime is associated with an unknown turbulent-like downscale energy flow. However, in equatorial regions subject to the equatorial wave guide, we have very different signatures. Currently we believe that in the eastern Pacific we have found a signal of westerly propagating Rossby waves which could also be a signal for the formation of storm clusters forming tropical cyclones. Then, in the western Pacific we have a signal that these have turned into tropical depressions, which would in turn typhoons.

New work on rotating flows is starting with a mathematics post-grad A Parmar to understand the experiments of P. Thomas in Engineering and P. Davidson at Cambridge.

References

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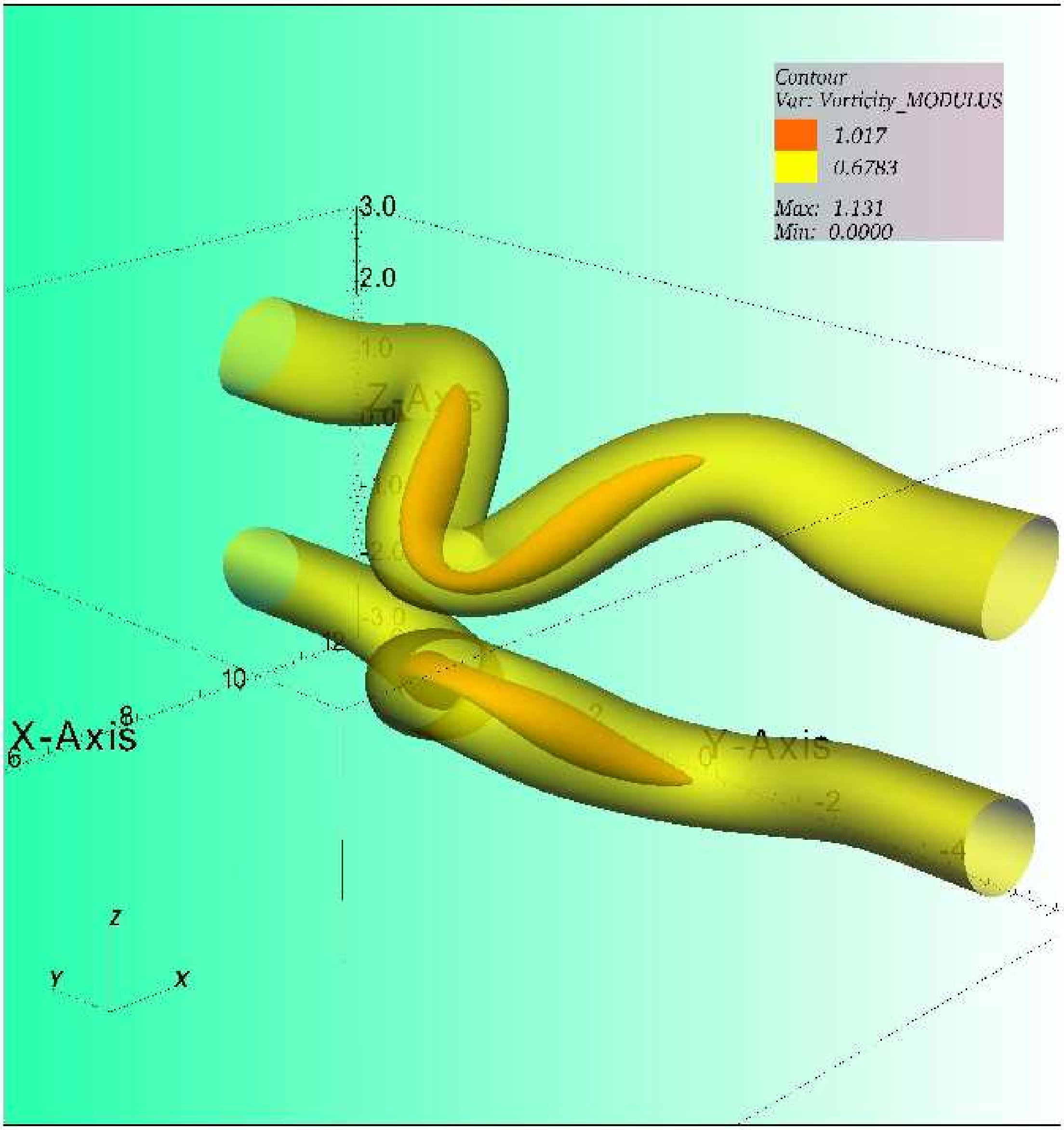

Figure 1: Euler anti-parallel vortices in full periodic domain near $t = 2.51$. Bright (yellow online) tubes are isosurface contours of vorticity modulus corresponding to 60% of the instantaneous maximum of vorticity modulus. Dark (red online) elongated blobs are isosurfaces corresponding to 90% of the maximum of vorticity modulus. $1024 \times 256 \times 2048$ points are used.

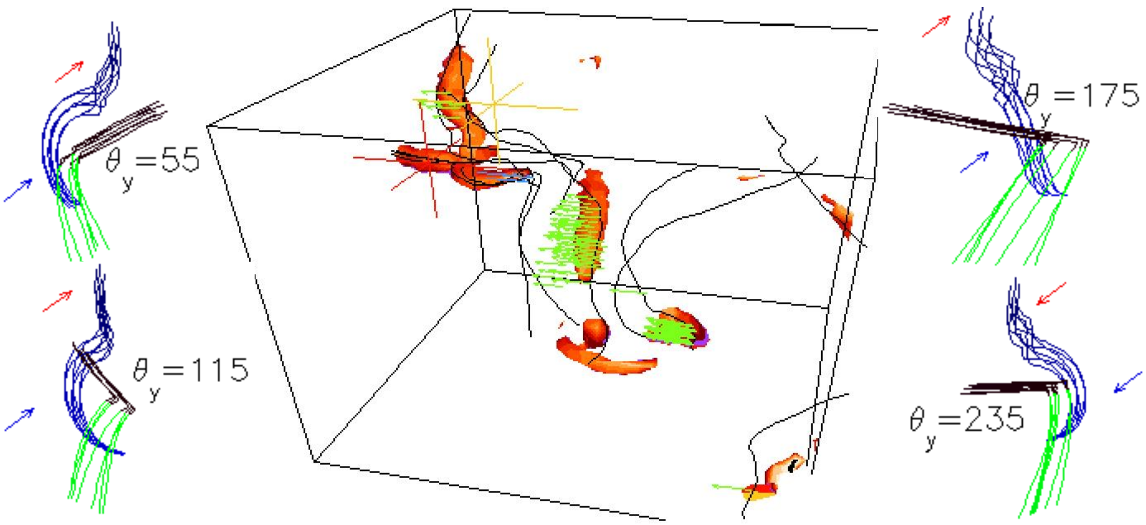

Figure 2: Central figure: isosurfaces of vorticity $|\omega|$ and blue/green hashes for ± concentrations of helicity $(\mathbf{u} \cdot \omega)$, demonstating that the tubular vortex regions after reconnection still have helicity associated with them. The surrounding diagrams represent how the reconnected vortices weave around each other, with blue/green representing the residual helicity.

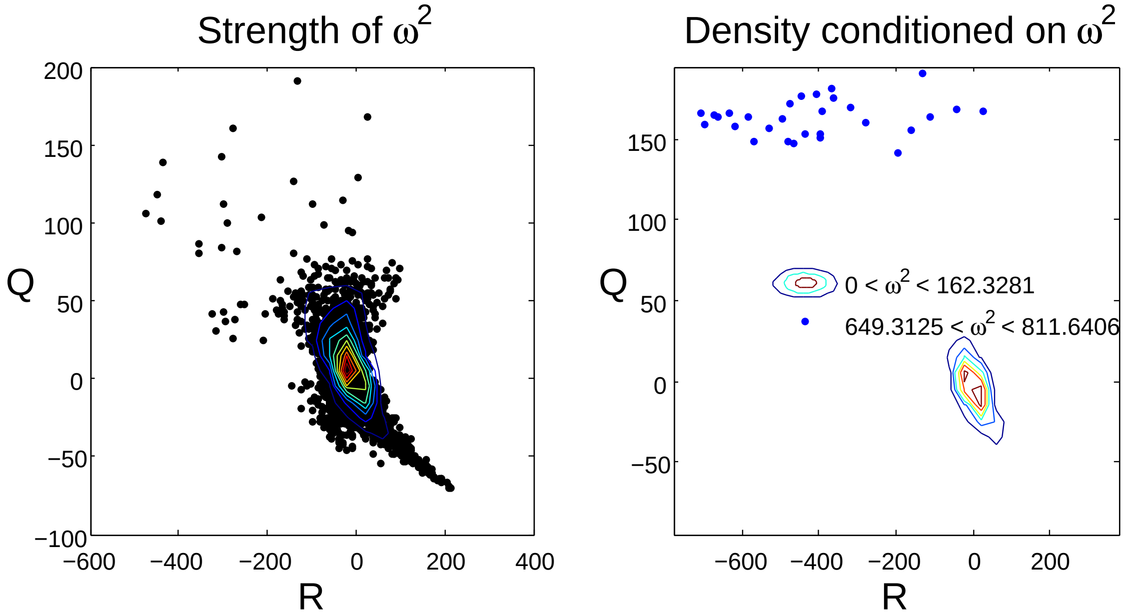

Figure 3: In the post-reconnection region and time of Fig. 2 points scattered in the second-order Cayley-Hamilton stress invariant $Q = -0.5(\text{strain})^2 + 0.25(\text{vorticity})^2$ (vertical) versus R (horizontal) the third-order Cayley-Hamilton invariant. The prediction is that points will congregate along the curve $R = (-\frac{4}{27}Q^3)^{1/2}$ in the lower right, which is observed. However, this is associated with the points with minimum vorticity (contoured on right). True singular dynamics should be associated with the points with maximum vorticity, that are in blue and located far from this curve.