A novel technique for the design of QMF filter banks with approximately linear phase, based on allpass filters

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Abstract

The two-channel QMF filter bank based on allpass sections is one of the best known circuits for building up a multi-channel filter bank for signal compression. An analysis-synthesis combination can satisfy two of the three PR conditions. The third, phase condition, can be met to any desired accuracy. In this contribution, we consider the development of explicit formulae for the coefficients of such filters.

1 Introduction

The two-channel allpass-based QMF bank is shown in Figure 1. It consists of two analysis and two synthesis filters. The \(2N\)th order real allpass subfilter has a transfer function \(G(z)\), given by the following:

\[
G(z) = \sum_{v=0}^{N} a_v z^{-2v} \sum_{u=0}^{N} a_u z^{-2u}
\]  

(1)

The corresponding phase function can be expressed as follows:

\[
\phi(\omega) = -2N\omega + 2\arctan \left( \frac{\sum_{v=0}^{N} a_v \sin 2n\omega}{1 + \sum_{v=0}^{N} a_v \cos 2n\omega} \right)
\]  

(2)

It has been known for some time that the parallel combination of two allpass subfilters(PCAS) has many desirable properties for the design of efficient filters used singly and in filter banks\([1 - 6]\). In particular, it is possible to simultaneously meet both magnitude and phase (or delay) specifications\([2,3]\). The complex case has been considered in several papers and leads to efficient solutions of the design problem, particularly where \(N\) is even\([7 - 9]\). In this contribution the real case will be considered. It is well known that the magnitude, phase and group delay of PCAS filters in the lowpass/bandstop case are given by the following expressions\([3]\):

\[
\begin{align*}
\Lambda(\omega) &= \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \\
\Phi(\omega) &= \pm(\phi_1 + \phi_2) \\
\tau(\omega) &= \pm(\tau_1 + \tau_2)
\end{align*}
\]

(3)

where \(\phi_1, \phi_2\) are respectively the phase of the upper and lower branch allpass subfilters. Similar expressions exist for the highpass/bandpass case\([3]\). The basic structure is shown in Fig. 1 where the sum and difference outputs are respectively the lowpass/bandstop and highpass/bandpass filtered inputs.

Figure 1: Two-channel allpass-based QMF bank

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2 Main principles

Rearranging eqn.(2) so that we have the following expression
\[
\sum_{n=1}^{N} a_n \sin 2n\omega = \tan \left[ \left( \phi(\omega) + 2N\omega \right) \right]
\]
(4)

where \( \phi(\omega) \) is the desired phase response for the upper or lower branch allpass filter and \( 2N \) is the filter order. Eqn.(4) now defines an approximation problem to determine the coefficients \( \{ a_n \} \) such that function on the left hand side approximates that on the right hand side.

Alternatively, if we express the tangent function in terms of sine and cosine functions we have,
\[
\sum_{n=1}^{N} a_n \sin 2n\omega = \sin \left[ \left( \phi(\omega) + 2N\omega \right) \right]
\]
(5)

Equating the numerators(or denominators) of eqn.(5) gives the following,
\[
\sum_{n=1}^{N} a_n \sin 2n\omega = \lambda(\omega) \sin \left[ \left( \phi(\omega) + 2N\omega \right) \right]
\]
(6)

and
\[
\sum_{n=1}^{N} a_n \cos 2n\omega = \lambda(\omega) \cos \left[ \left( \phi(\omega) + 2N\omega \right) \right]
\]
(7)

We can combine eqns.(6) and (7) into the complex exponential form as follows,
\[
\sum_{n=1}^{N} a_n \exp(j2n\omega) = \lambda(\omega) \exp \left[ j \left( \phi(\omega) + 2N\omega \right) \right]
\]
(8)

where \( \lambda(\omega) \) is an arbitrary real function of frequency.

Various design strategies are possible involving eqn.(6),(7) or(8). In this paper we consider eqn.(6) to determine the coefficients with \( \lambda(\omega) \) set to 1. We can use the fact that the functions \( \{ \sin 2n\omega \}, n = 1, \ldots, N \) are an orthogonal set over the range \([0,\pi]\) as the phase is an odd function. Multiplying both sides of eqn.(6) by \( \sin 2m\omega \) and integrating over this range gives,
\[
a_n = \frac{1}{\pi} \int_{0}^{\pi} \sin \left[ \left( \phi(\omega) + 2N\omega \right) \right] \sin 2n\omega \, d\omega
\]
(9)

For the lower branch, the delay is incorporated into the design process. Consequently the first term in the integrand becomes \( \sin \left( \phi(\omega) + 2N\omega + \omega \right) \).

3 Phase function specification

The usefulness of eqn.(9) depends on the availability of a suitable phase function. In the paper by Lu et al[10], it was suggested that the ideal phase responses for the upper and lower branches should be linear but splitting in the transition band in such a way that there is a phase difference of \( \pi \) in the stopband. Moreover the phase of the highest order branch should be \( \pi/2 \) below the average phase whilst that of the other branch should be \( \pi/2 \) above. A typical set of responses is shown in Figure 2 so as to provide good magnitude and approximately linear phase(ALP) characteristics of the QMF filters. A piecewise linear definition of these phase functions can be found easily.

![Figure 2: Typical phase responses of upper branch(∗), lower branch(+) and overall(solid line).](image-url)

For the upper(lower) branch the piecewise linear definition is as follows,
\[
\phi(\omega) = \begin{cases} 
- k\omega & 0 \leq \omega \leq \omega_p \\
- k\omega + \frac{\pi}{2} \left( \omega - \omega_p \right) & \omega_p \leq \omega \leq \omega_s \\
- k\omega + \frac{\pi}{2} \left( \omega - \omega_s \right) & \omega_s \leq \omega \leq \pi \\
\end{cases}
\]
(10)

where \( k \) is the phase slope or the group delay in the pass and stopbands.

In this contribution, various simplifications are used in order to show the technique clearly but they are not necessary for the technique to work. The branch orders differ only by the delay in the lower branch. Also
\[ k = 2N + \frac{1}{4} \text{[10]}. \] The passband and stopband edges are symmetric around \( f_s/4 \). Thus \( \omega_p + \omega_s = \pi \). We also define \( \Delta = \omega_s - \omega_p \) so that \( \Delta + \pi = 2\omega_p \), which is useful in simplifying expressions after integration.

\[
a_n = \frac{(-1)^n}{2\Delta} \left\{ \frac{1}{(2n + \frac{1}{4})(2n + \frac{3}{4} - \frac{1}{2\pi})} \sin \left( \frac{\omega_p - n\Delta}{4} \right) - \frac{1}{(2n - \frac{1}{4})(2n - \frac{3}{4} + \frac{1}{2\pi})} \sin \left( \frac{\omega_s + n\Delta}{4} \right) \right\} \] (11)

The coefficients of the lower branch can be found in a similar way from the following integration,

\[
b_n = \frac{2}{\pi} \int \sin \left[ \phi(\omega) + 2\pi p \omega + \omega \right] \sin 2\pi p \omega d\omega \] (12)

noting that the ideal phase uses the lower signs in eqn.(10).

Because of all the various assumptions discussed in the previous section, it turns out that \( b_n = -a_n \).

5 Design example

To design a QMF pair requires only the selection of the filter order and passband edge. Let us select an allpass filter order of 4 so that the QMF filters are \( 9^\text{th} \) order. Let the passband edge frequency, \( f_p = 0.2 \) then \( f_s = 0.3 \) and \( \Delta = 2\pi \times 0.1 \).

Applying eqns.(11) and (12) gives the following sets of coefficients:

\( a_0 = 1, \quad a_1 = -0.23192 \quad \text{and} \quad a_2 = 0.092935 \quad \text{and} \quad b_0 = 1, \quad b_1 = 0.23192 \quad \text{and} \quad b_2 = -0.092935 \)

The analysis of the QMF filters was carried out and the phase and magnitude are shown in Figures 3 and 4. It can be seen that the overall phase has high linearity.

Figure 3: Phase response of upper branch(*), lower branch(+) and overall(solid line) for \( 9^\text{th} \) order example.
6 Conclusions

This paper has described a novel technique to design a two-channel QMF filter bank which can ultimately be used as the building block for the multi-channel case. The technique exploits the fact that the phase of each subband can be split into two parts and, as a consequence, the orthogonal nature of sine and cosine functions can be applied. This leads to explicit formulae for the coefficients in the QMF case. The technique is more general because different phase functions can be approximated and explicit formulae will result so long as the resulting functions are integrable. In addition there are three basic design approaches because the numerator, denominator or both may be used to derive the coefficients. More research is required to consider the error in the methods and to determine a link between the specification and the phase definition.

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References


