ES440: CFD

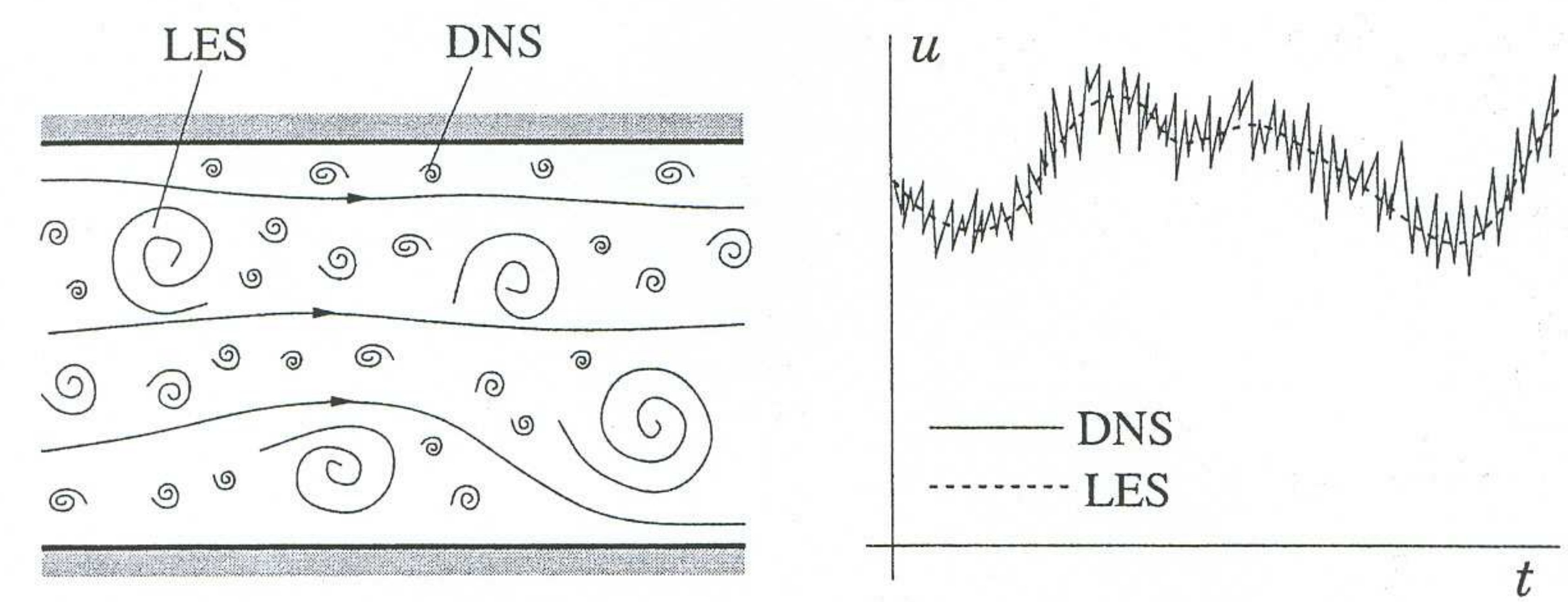
Chapter 9. Turbulent Flows

Dr Yongmann M. Chung
http://www.eng.warwick.ac.uk/staff/ymc/ES440.html
Y.M.Chung@warwick.ac.uk

School of Engineering & Centre for Scientific Computing
University of Warwick

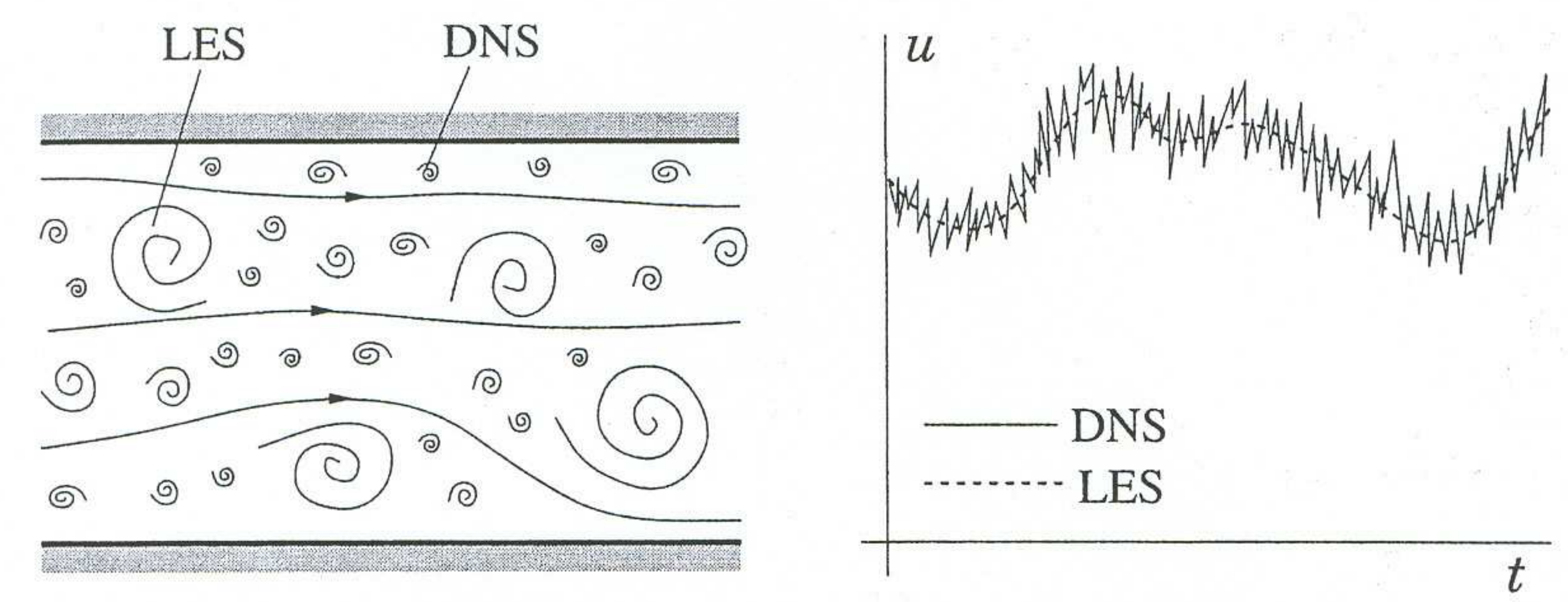
Chapter 9
Turbulent Flows

Turbulent Flows



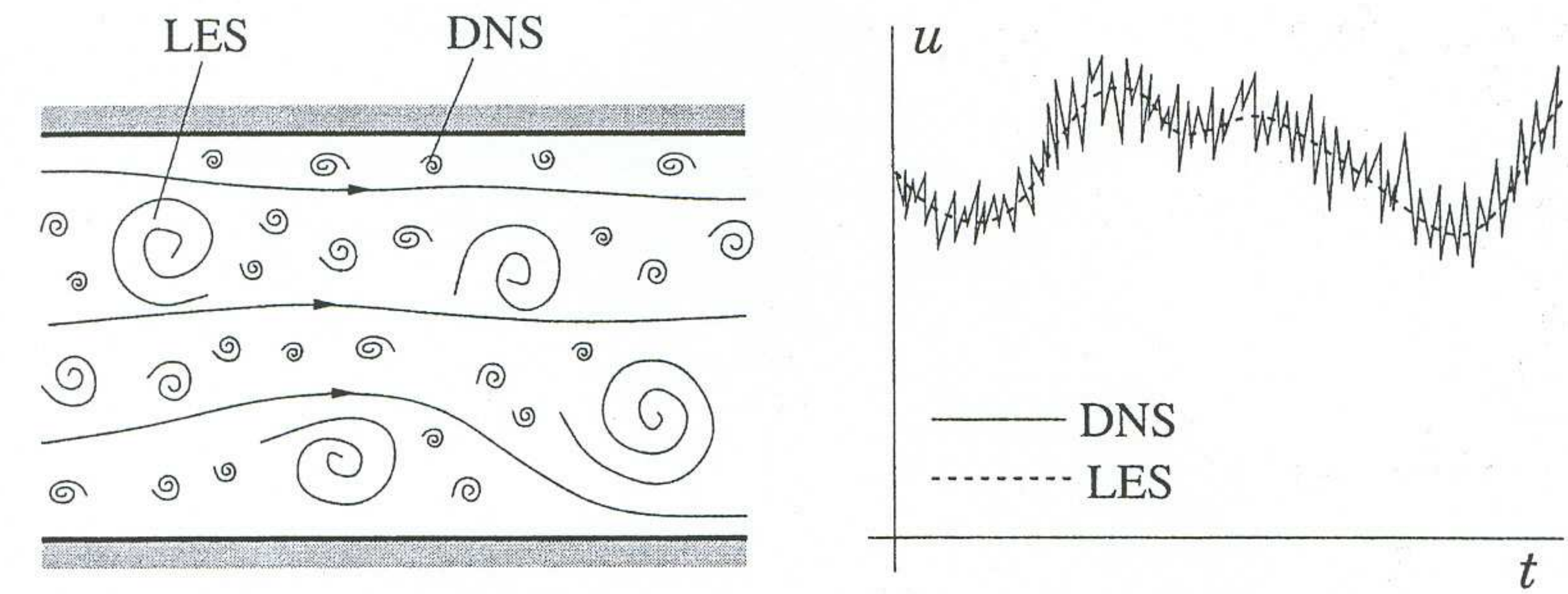
- Turbulent flow is ***Unsteady*** & complex.
- ***Small-scale*** motions as well as large-scale motions.
- DNS/LES/RANS

DNS of Isotropic Turbulence



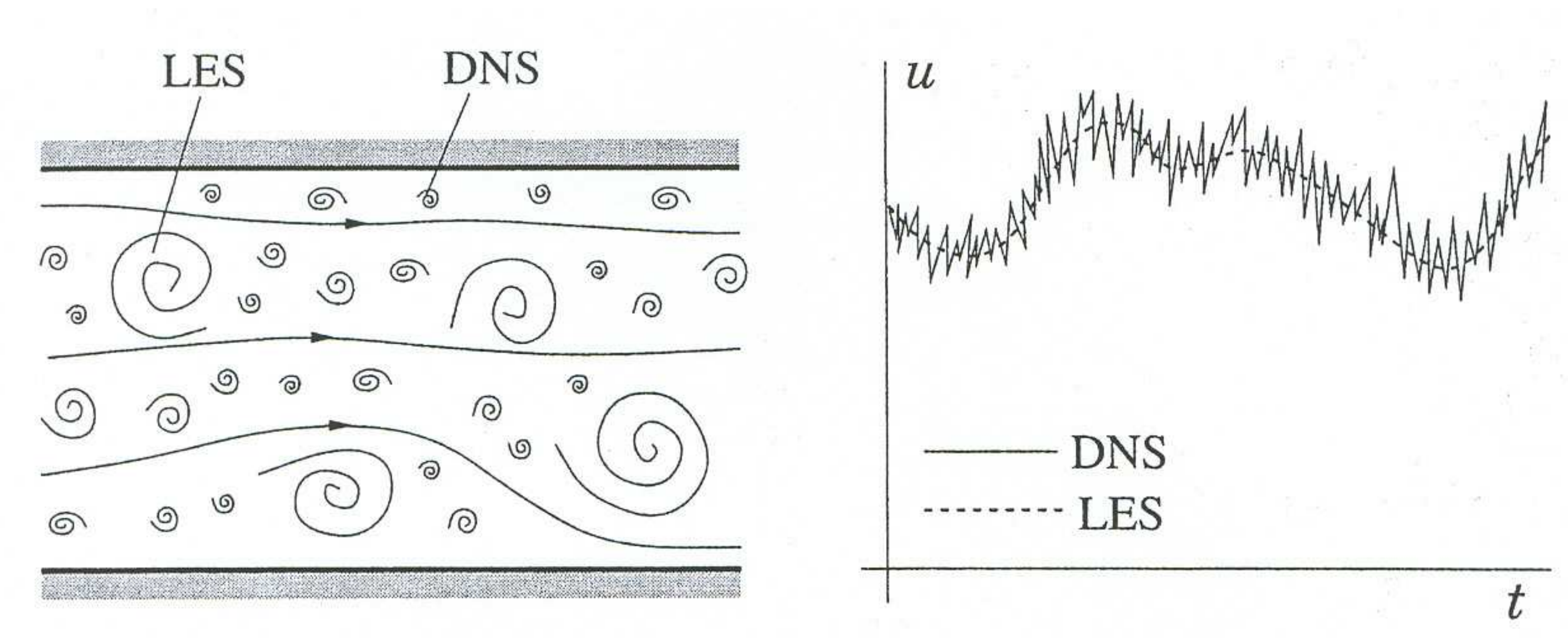
- Direct Numerical Simulation of the Navier-Stokes equations with no models.
- Small grid spacings Δx_i & time steps Δt.
- Needs large computing power.

DNS of Isotropic Turbulence - Cont'd



- Largest scale: ***Integral length Scale***, $l = q^3/\epsilon$.
- Smallest scale: ***Kolmogorov length scale***, $\eta = (\nu^3/\epsilon)^{1/4}$
- The ratio between the largest and the smallest scales is $\dfrac{l}{\eta}$

DNS of Isotropic Turbulence - Cont'd 3

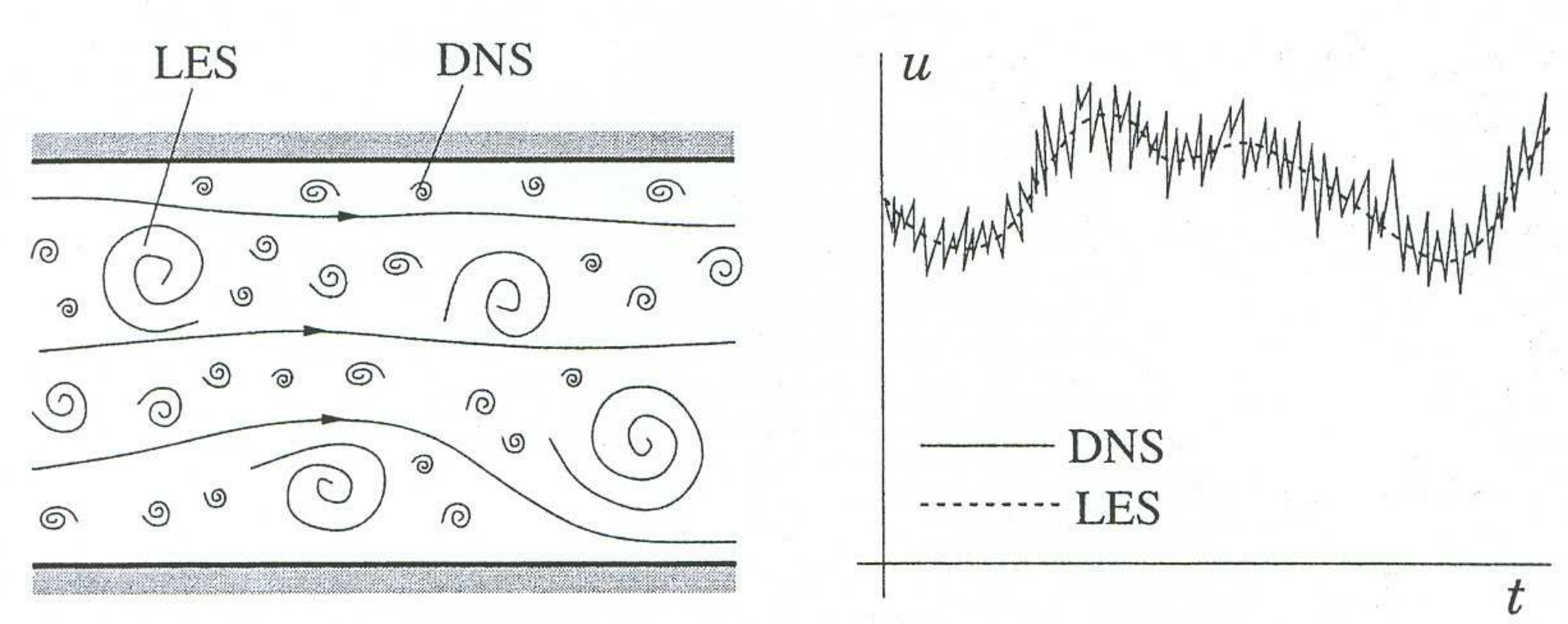


- $Re = q^4/(\nu\epsilon)$ is the Reynolds number of the large-scale turbulence.

$$\frac{l}{\eta} = \frac{q^3/\epsilon}{(\nu^3/\epsilon)^{1/4}},$$

$$\frac{l}{\eta} = \frac{q^3}{(\nu^3\epsilon^3)^{1/4}} = Re^{3/4}.$$

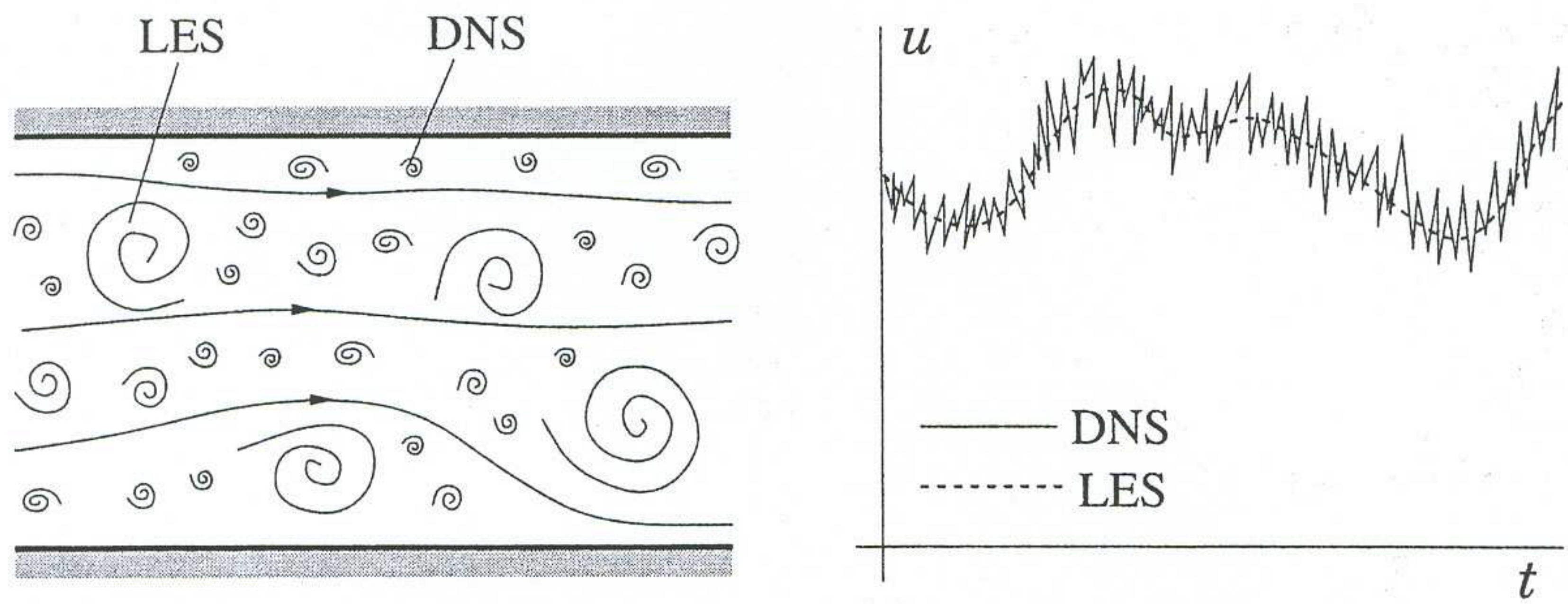
DNS of Isotropic Turbulence - Cont'd 4



- The ratio between the largest and the smallest scales is
  $\frac{l}{\eta} = Re^{3/4}$
- The number of grid points required is therefore

$$N_{xyz} = \left(\frac{L}{\Delta}\right)^3 \approx \left(\frac{l}{\eta}\right)^3 = Re^{9/4}.$$

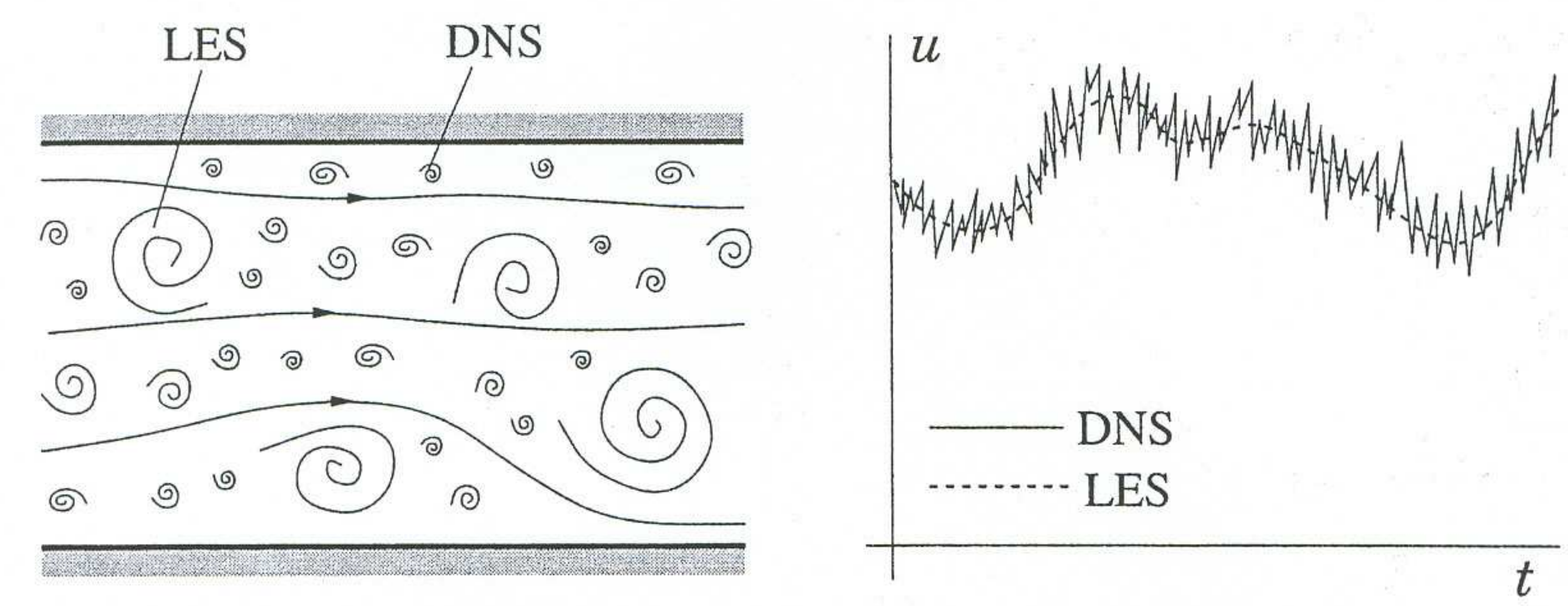
DNS of Wall Turbulence



- The ratio of the channel width to the scale of the smallest eddies is: $\dfrac{L}{\eta} \approx Re^{0.9}$
- So, the number of grid points required in one-dimension is

$$N_{xyz} = \left(\frac{L}{\Delta}\right)^3 \approx \left(\frac{L}{\eta}\right)^3 = Re^{2.7}. \quad (1)$$

LES



- Large-Eddy Simulation.
- Resolve only large-scale motions: $u = \overline{u} + u'$.
  - $\overline{u}$ is the large-scale velocity & u' is the small-scale velocity.
- Use sub-grid scale (SGS) models for small-scale motions.

LES - Cont'd

- The large-scale field is defined with the aid of the filter function G:

$$\overline{u}(x) = \int_D G(x - x')u(x')dx', \tag{2}$$

- Continuity equation:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0, \tag{3}$$

- Navier-Stokes equation:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u_i u_j}\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j},$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\tau_{ij} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j}.$$

LES - Cont'd 3

- Governing equations for ***large-scale*** motions:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\tau_{ij} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} .$$

- All the terms in the above equations are resolved in the LES except the residual stress tensor

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j . \tag{4}$$

- In the most commonly used model, developed by Smagorinsky, the eddy viscosity model is obtained by assuming that the small scales are in equilibrium, so that energy production and dissipation are in balance. This yields an expression of the form

$$\tau_{ij} - 1/3\delta_{ij}\tau_{kk} = -2\nu_t S_{ij} . \tag{5}$$

LES - Cont'd 4

- Smagorinsky model:

$$\tau_{ij} - 1/3\delta_{ij}\tau_{kk} = -2\nu_t S_{ij}. \tag{6}$$

-

$$\nu_t = (C_s\Delta)^2 \, |\overline{S}|, \tag{7}$$

where C_s is the Smagorinsky constant, and Δ is the filter width, $|\overline{S}| = \sqrt{2S_{ij}S_{ij}}$ is the magnitude of large-scale strain-rate tensor

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right). \tag{8}$$

LES - Cont'd 5

- Smagorinsky model:

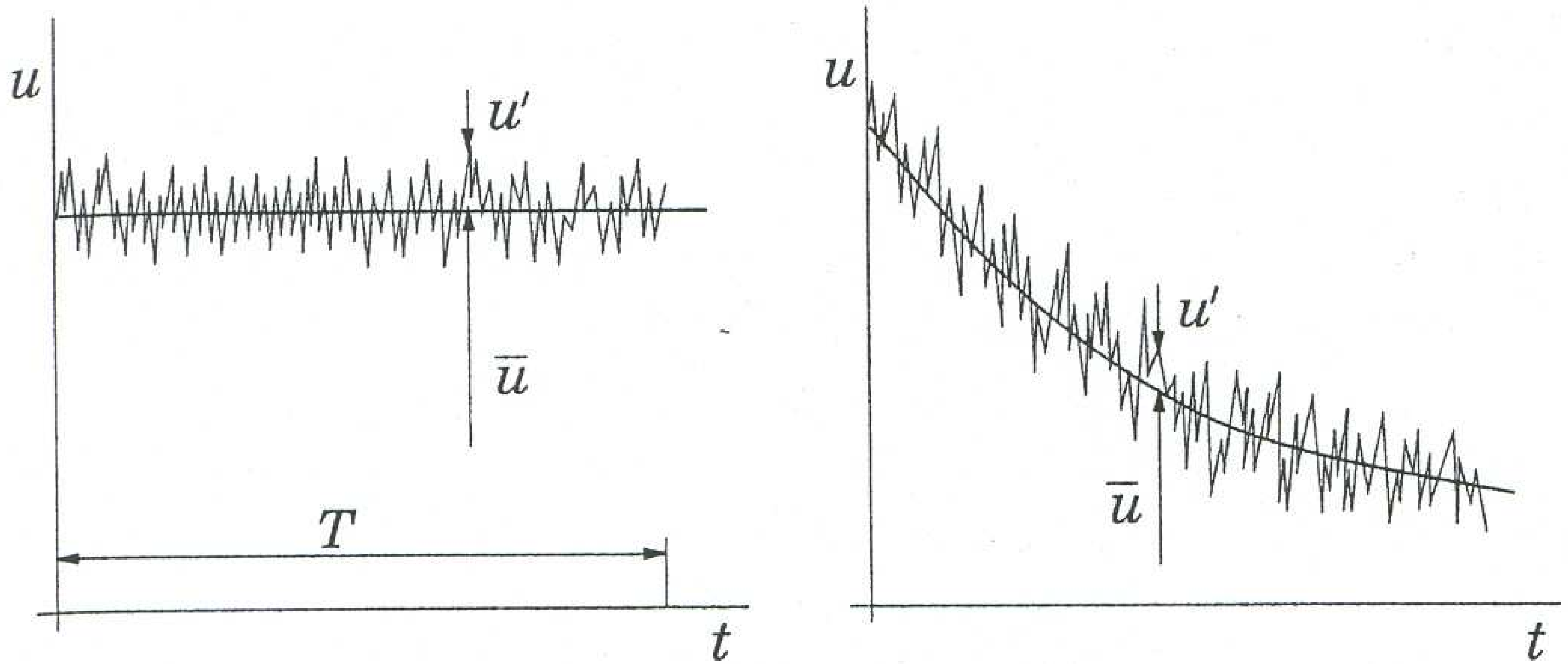
$$\tau_{ij} - 1/3\delta_{ij}\tau_{kk} = -2\nu_t S_{ij}, \tag{9}$$

$$\nu_t = (C_s\Delta)^2 \, |\overline{S}|, \tag{10}$$

- The filter width, Δ, represents the characteristic length scale of the largest subgrid-scale eddies, which are unresolved in the filtered momentum equations. Δ was defined as the geometric mean of the unidirectional filter width, i.e.,

$$\Delta = (\Delta_1\Delta_2\Delta_3)^{1/3}. \tag{11}$$

RANS



- Reynolds-Averaged Navier-Stokes Simulation.
- Reynolds Averaging: $u = \overline{u} + u'$
  - $\overline{u}$ is the time-averaged velocity & u' is the fluctuations.
- Turbulence models for all turbulent motions.

RANS - Cont'd

- The large-scale field is defined with the aid of the filter function G:

$$\overline{u}(x) = \frac{1}{T}\int_0^T u(x,t)dt, \tag{12}$$

- Continuity equation:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0, \tag{13}$$

- Navier-Stokes equation:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u_i u_j}\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j},$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\tau_{ij} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j}.$$

RANS - Cont'd 3

- Turbulence models:

$$\tau_{ij} - 1/3\delta_{ij}\tau_{kk} = -2\nu_t S_{ij}, \tag{14}$$

$$\nu_t = C_\mu q l, \tag{15}$$

  - C_μ is a dimensionless constant.
  - q is the velocity scale.
  - l is the length scale.

$$\nu_t = C_\mu \sqrt{k} l = C_\mu \frac{k^2}{\varepsilon}, \tag{16}$$