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Example Questions

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1. Classify the following PDEs:

   (a) Laplace's equation,

   (b) Wave equation,

   (c) Unsteady heat equation,

   (d) Incompressible Navier-Stokes equations,

   (e) Boundary-layer equations,

   (f) Euler equations,

2. Using Taylor's series, derive the finite difference approximation of the following derivatives using the method and order indicated.

   (a) $\dfrac{\partial f}{\partial x}$, forward difference, $O(\Delta x)$,

   (b) $\dfrac{\partial f}{\partial x}$, backward difference, $O(\Delta x)$,

   (c) $\dfrac{\partial f}{\partial x}$, central difference, $O(\Delta x^2)$,

   (d) $\dfrac{\partial f}{\partial x}$, central difference, $O(\Delta x^4)$,

   (e) $\dfrac{\partial^2 f}{\partial x^2}$, central difference, $O(\Delta x^2)$.

3. Establish the truncation error (T.E.) of the following finite-difference approximation to $\dfrac{\partial f}{\partial x}$ at the point (i, j) for a uniform mesh.

$$\frac{\partial f}{\partial x} = \frac{f_{i+1} - f_{i-1}}{2\Delta x}. \tag{1}$$

4. Use the modified wavenumber analysis to assess the accuracy of the central difference formula

$$\frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2}. \tag{2}$$

5. Transform the steady 2-D incompressible equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3}$$

to (ξ, η) computational space using the transformation

$$\xi = x, \quad \eta = \frac{y}{x^2}. \tag{4}$$

6. Transform Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0. \tag{5}$$

to (ξ, η) computational space using the transformation

$$\xi = x, \quad \eta = g(y). \tag{6}$$

7. Derive the finite difference equation of Poisson equation with central difference schemes in FDM or FVM.

8. Summarise the following CFD solution procedures:

   (a) ***Step 1***: Problem Definition - Governing equations.

   (b) ***Step 2***: Computation Domain.

   (c) ***Step 3***: Boundary Conditions.

   (d) ***Step 4***: Computational Grid.

   (e) ***Step 5***: Computer Simulation.

   (f) ***Step 6***: Results & Discussion - Validation, Comparison & Analysis.

9. How to determine the optimal computational domain size (**Step 2**) and the number of grid points (**Step 3**) when the largest length scale (*integral length scale*) is $1m$ and the smallest length scale (*Kolmogorov length scale*) is $1mm$?

10. Define the following terms in your own words:

    (a) FDM

    (b) FVM

    (c) Numerical error

    (d) Truncation error

    (e) Convergence

    (f) Iterative method

(g) Modified wavenumber

(h) TDMA

(i) ADI

(j) Crank-Nicolson

(k) Unsteady problem

(l) Explicit and implicit methods for unsteady problem

(m) CFL condition

(n) Finite difference equation

(o) Elliptic, parabolic, and hyperbolic types of PDEs

(p) Fractional step method

(q) Turbulent flow

(r) DNS, LES, and RANS

Good Luck!