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# The Automatic Analysis of Interferometric Data - FRAN

Description and User Guide

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## **1. Introduction.**

Interferometry is a successful method of measuring fields of refractive index and displacement. It therefore has many applications in the measurement of fluid flows, photo elastic stresses, surfaces and vibrational analysis. The measurements are encoded within a fringe pattern, figure 1, where the fringes represent lines of constant phase. By extracting the phase information from the fringe pattern a quantitative height map is produced. The height map may represent a surface profile, a displacement, or the optical path difference through a phase object. In practice obtaining this height map is difficult due to ambiguities in the fringe data, speckle noise and the resolution of the imaging system.

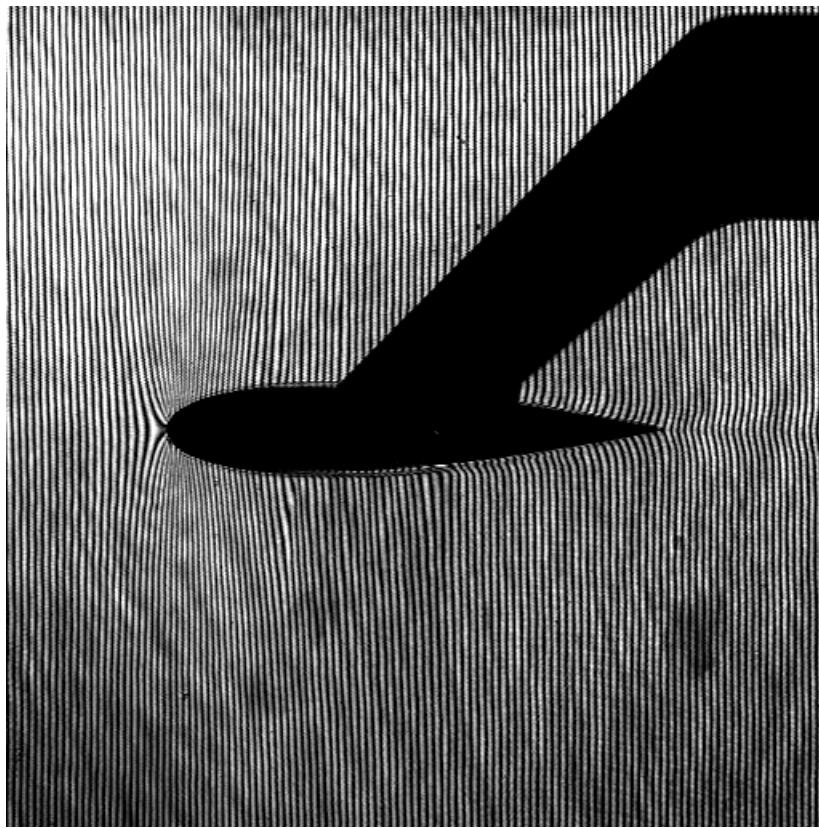


Figure 1. Interference fringes round an aerofoil. Picture taken by Steve Parker at BAe.

Described here are two methods of extracting the encoded phase information, each with differing merits and drawbacks. In each case the phase information is presented as tan fringes, figure 2, the discontinuous function removing a directional ambiguity. A largely noise immune approach to ‘unwrapping’ the phase information, ‘wrapped’ within a modulo of  $2\pi$  into a continuous surface, figure 3, is also described.

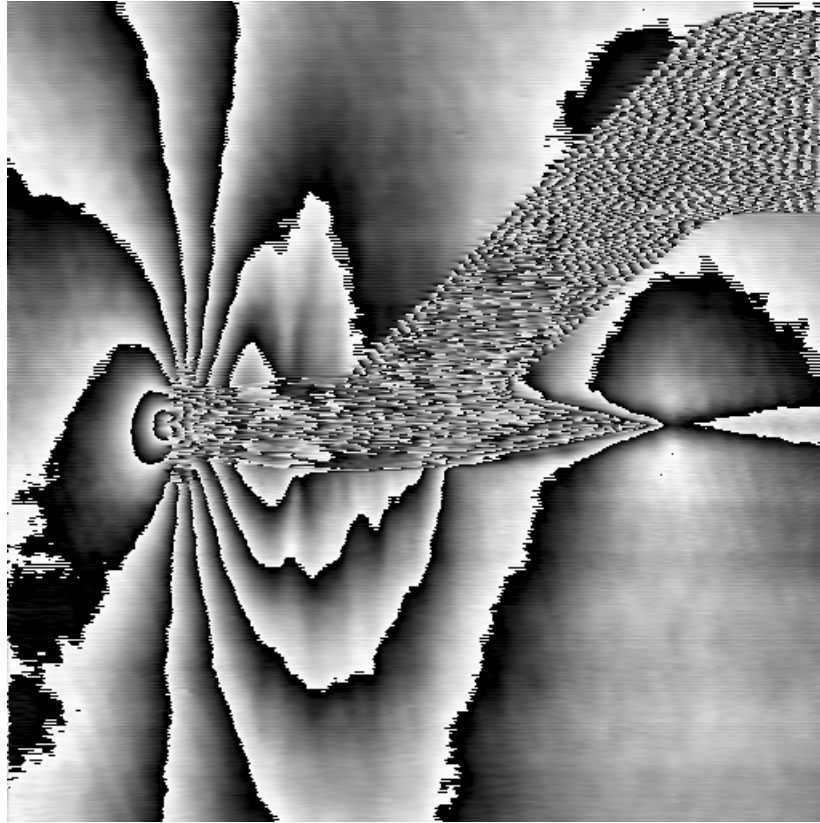


Figure 2. The wrapped phase map showing the tan fringes.

Both the Fourier transform and phase stepping methods of fringe pattern analysis are successful means of extracting phase information from linear fringe patterns generated by the interference of tilted wave fronts. Figures 4 and 5 show alternative ways of achieving such fringe patterns. The first shows the use of a back silvered mirror to generate multiple tilted wave fronts. The second figure, the dual reference approach, allows the phase shift between the beams to be varied. By increasing the angle of tilt the frequency of the fringes, the carrier frequency, can be increased. This allows the carrier frequency to be adjusted so that the phase information to be encoded by the deformation of the fringes is a slowly varying function in comparison. If the phase change between fringes is greater than  $2\pi$  a local discontinuity will result as a phase change of  $\pi$  can not be distinguished from  $n\pi$ .

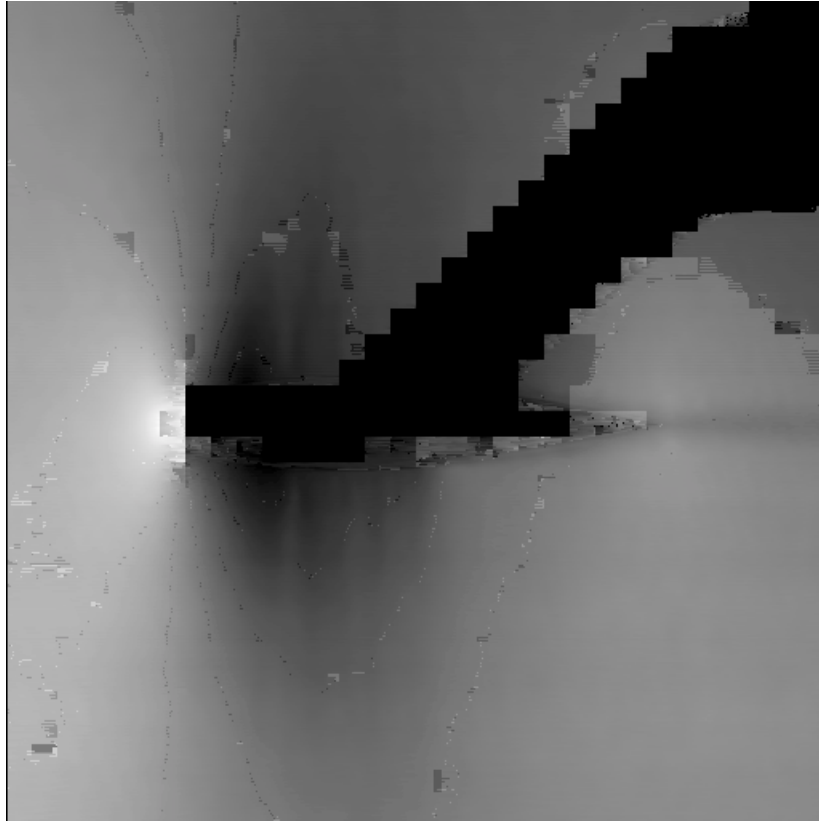


Figure 3. The unwrapped, continuous phase map.

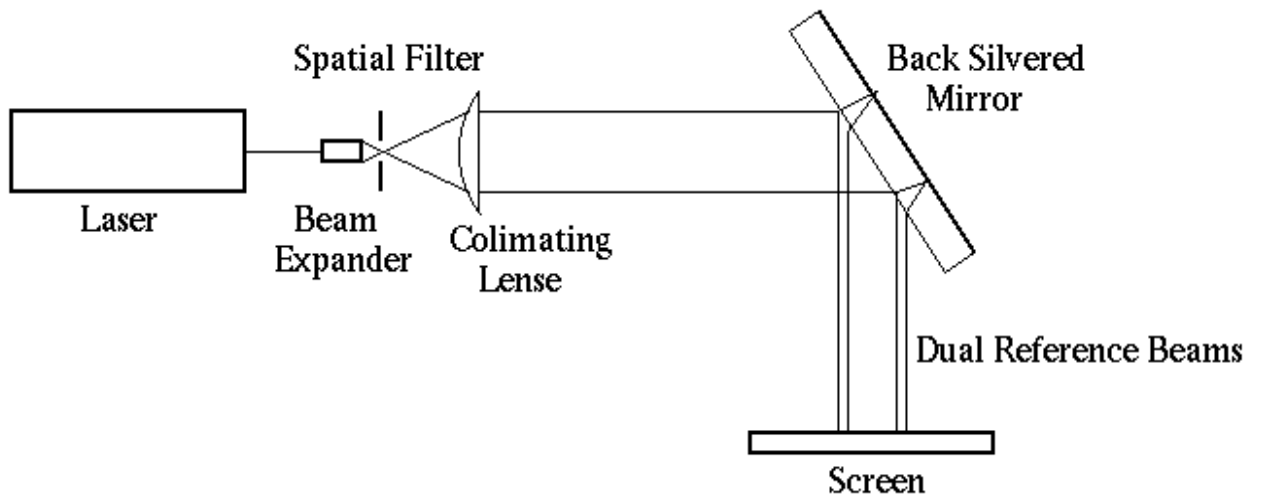


Figure 4. A Shearing Interferometer used to generate linear fringe patterns.

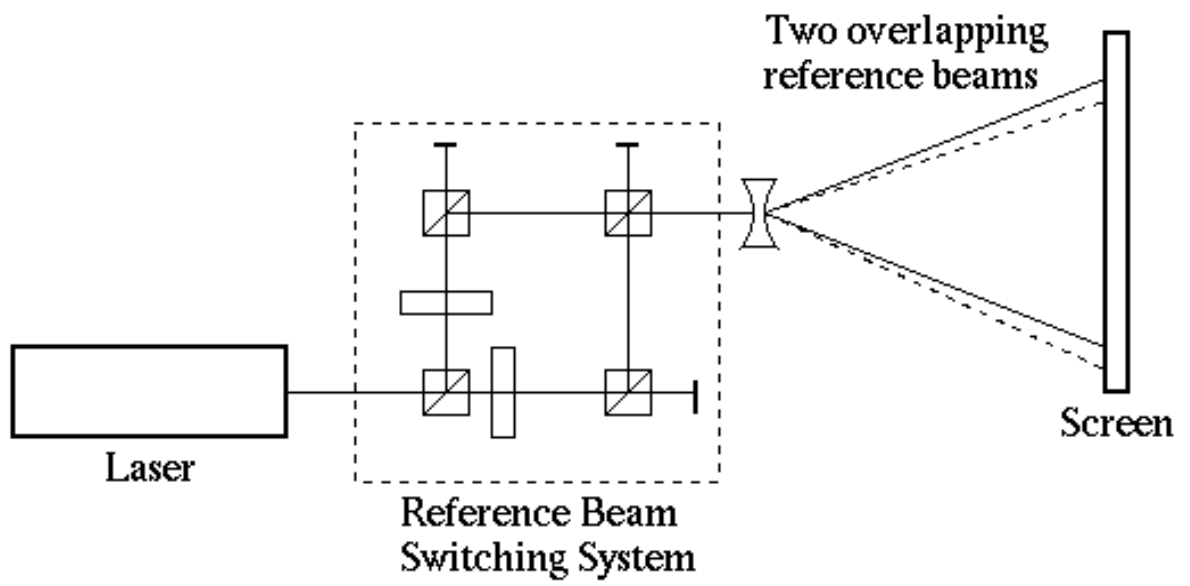


Figure 5. A Dual Reference interferometer used in phase stepping.

## 2. The Fourier Transform Method of Fringe Analysis <sup>[1],[2],[3]</sup> .

The phase information is encoded into the interferogram in the form of bending of the fringes. As the fringes are deformed the uniform spacing between the fringes is lost, generating localised areas of compressed and expanded fringe spacing. Scanning across these areas would reveal changes in the fringe frequency corresponding to the change in fringe spacing. By extracting this fringe frequency from the dominant carrier frequency the required phase information is isolated. This is achieved by using a Fourier transform <sup>[1]</sup> , equations 2.1 and 2.2 giving the Fourier and inverse Fourier transforms respectively.

$$G(u) = \int_{-\infty}^{\infty} g(x) e^{-j2\pi ux} \quad (2.1)$$

$$g(x) = \int_{-\infty}^{\infty} G(u) e^{j2\pi ux} \quad (2.2)$$

The discrete Fourier transform is more suitable to computational methods, the discrete transform pair being defined as:

$$G(n) = \frac{1}{N} \sum_{k=0}^{N-1} g(k) e^{-\frac{j2\pi nk}{N}} \quad (2.3)$$

$$g(k) = \sum_{n=0}^{N-1} G(n) e^{\frac{j2\pi nk}{N}} \quad (2.4)$$

where  $n=0,1,2,3,\dots,N-1$ .

The interferogram can be described by, <sup>[1], [2]</sup> :

$$g(x,y) = a(x,y) + b(x,y) \cos[2\pi f_0 x + \phi(x,y)] \quad (2.5)$$

where  $a(x,y)$  is the background noise and  $b(x,y)$  the modulation noise.  $f_0$  is the carrier frequency in the x direction and  $\phi(x,y)$  is the phase information of interest.

Equation 2.5 is written in its complex form as follows:

$$g(x,y) = a(x,y) + 1/2 b(x,y) (\cos [2\pi f_0 x + \phi(x,y)] + j \sin[2\pi f_0 x + \phi(x,y)] \\ + \cos[2\pi f_0 x + \phi(x,y)] - j \sin[2\pi f_0 x + \phi(x,y)]) \quad (2.6)$$

where  $\cos[2\pi f_0 x + \phi(x,y)] - j\sin[2\pi f_0 x + \phi(x,y)]$  is the complex conjugate.

Equation 2.6 can be written in a more convenient form thus

$$g(x,y) = a(x,y) + b(x,y)/2 (e^{j\phi(x,y)} e^{j2\pi f_0 x} + e^{-j\phi(x,y)} e^{-j2\pi f_0 x}) \quad (2.7)$$

Letting  $c(x,y) = b(x,y)e^{j\phi(x,y)}/2$  and  $c^*(x,y)$  its complex conjugate equation 2.7 becomes:

$$g(x,y) = a(x,y) + c(x,y)e^{j2\pi f_0 x} + c^*(x,y)e^{-j2\pi f_0 x} \quad (2.8)$$

To apply a discrete Fourier transform on the intensity distribution the signal needs to be periodic. To achieve this the ends of each raster of pixels in the x - direction are brought to zero using a Papoulis window<sup>[2]</sup>, figures 6 and 7.

Figure 6. Intensity data of one raster of pixels.

Figure 7. Intensity data weighted by a Papoulis window.

The Fourier transform of the intensity distribution is given by

$$G(f,y) = A(f,y) + C(f-f_0,y) + C^*(f+f_0,y) \quad (2.9)$$

The power spectrum of the raster consists of two spectral side lobes corresponding to the second and third terms of equation 2.9, centred around the carrier frequency, figure 8.

Figure 8. Power spectrum of raster with carrier frequency and deformation.

Figure 9. Side lobe after being weighted by the Hanning window and translated by the carrier frequency to the origin position.

Only one of the side lobes needs to be considered. It is desirable for  $\phi(x,y)$  to be slowly varying function compared to the carrier frequency. By choosing a carrier frequency that is suitable for the resolution of the image, say five pixels per fringe for a digital image, and carefully choosing the cut off frequencies, the phase information can be separated from the unwanted low frequency modulation and the high frequency speckle. By weighting one of the side lobes using a Hanning window<sup>[2]</sup>, figure 9, and

translating it to the origin by the carrier frequency,  $C(f,y)$  is obtained. The phase map is calculated using the inverse transform of  $C(f,y)$ . The phase distribution is given by

$$\phi(x, y) = \arctan \left[ \frac{\Im[c(x, y)]}{\Re[c(x, y)]} \right] \quad (2.10)$$

where  $\Im[c(x,y)]$  and  $\Re[c(x,y)]$  are the imaginary and real components of  $c(x,y)$  respectively. The use of the arctan function gives a discontinuous phase map wrapped in a modulo of  $2\pi$ . The discontinuous form of the arctan function removes the directional ambiguity.

### 3. The Phase Stepping Method of Fringe Analysis <sup>[3],[4],[5]</sup>.

With the FFT technique a sign ambiguity exists due to the nature of the fringes. This problem is overcome by deciding on a convention prior to the analysis and relies on the knowledge of the subject being sufficient. To overcome this and to increase accuracy the phase stepping technique can be used. The phase stepping technique uses either three or four phase shifted images of the subject. These images can each be described by a linear equation, summarised in equation 3.1.

$$I_{n\theta} = a(x,y) + b(x,y) \cos (\phi(x,y) + n\theta) \quad (3.1)$$

where  $I_{n\theta}$  is the intensity of the grey scale at a pixel in a digital image,  $a(x,y)$  and  $b(x,y)$  are unwanted speckle noise and modulation terms respectively,  $\theta$  is the phase step and  $\Delta\phi$  is the phase information of interest.  $n$  is an integer. To achieve the desired phase shift between successive images the apparatus described in figure 5 is required as it allows the phase to be varied between the reference beams. The apparatus in figure 6 can therefore only be used for the FFT technique unless the thickness of the mirror can be varied.

Typically the phase stepping mechanism is calibrated allowing the wrapped phase map to be evaluated using only three images. The phase is given by

$$\Delta\phi = \tan^{-1} \left[ \frac{(I_{3\theta} - I_{\theta})(\cos \theta - 1) - (I_{2\theta} - I_{\theta})(\cos 2\theta - 1)}{\sin \theta (I_{3\theta} - I_{\theta}) - \sin 2\theta (I_{2\theta} - I_{\theta})} \right] \quad (3.2)$$

The use of four images allows the phase step to be calculated so negates the requirement for time consuming mechanism calibration although the phase step has to be kept uniform between each fringe pattern. The phase step is given by

$$\theta = \cos^{-1} \left[ \frac{I_{\theta} - I_{2\theta} + I_{3\theta} - I_{4\theta}}{2[I_{2\theta} - I_{3\theta}]} \right] \quad (3.3)$$

## 4. Phase Unwrapping <sup>[2]</sup>.

For a perfectly consistent wrapped phase map, in the absence of noise, the field may be unwrapped by scanning across it, adding or subtracting  $2\pi$  every time an edge is reached. In practice inconsistencies and noise make this procedure an unrealistic method of phase unwrapping. A more robust method of phase unwrapping is to unwrap in a path that negotiates both spike noise and local area inconsistencies.

### 4.1. Minimum Spanning Tree Approach to Phase Unwrapping.

[2] describes the use of a minimum spanning tree as a mechanism for computing the consistency of possible phase unwrapping routes. An unwrapping path is constructed in order to maximise confidence, confidence being weighted according to a minimum spanning tree. At its lowest level paths between pixels are compared to circumvent spike noise. Neighbouring pixels to each individual pixel are considered, the path progressing to the pixel with the minimum phase change. As the unwrap path seeks to minimise the phase change at each step the spike noise, characterised by a rapid change in phase, is not considered until the end of an unwrap path. Its threat to the overall solution is therefore minimised.

At a higher level tiles of pixels are compared to deal with local area inconsistencies such as aliasing and natural inconsistencies. The aim is that areas of local inconsistencies are contained within the tiles and are dealt with in a similar way to spike noise. Inconsistencies are detected by comparing against the unwrapped edges of neighbouring tiles. Inconsistencies are avoided in the unwrap path. The tiles are made to overlap each other slightly to aid the comparison. In addition to this, edge detection algorithms are used to find whether there are fringes present within a tile. If no fringes are found then clearly the tile does not need to be unwrapped. This greatly reduces the time required for the unwrapping process, especially where large areas of none fringe information are included in the image such as silhouettes and boundaries.

By adopting this largely noise immune phase unwrapping strategy, a fringe pattern can be solved despite the presence of spike noise and local area inconsistencies. Their threat to the overall solution is minimised by only including them at the end of an unwrap path.

## **5. Conclusions.**

Both the Fourier transform and phase stepping techniques of fringe analysis are successful methods of extracting information from linear fringe patterns.

The Fourier transform technique requires only one fringe pattern in order to give this information. By carefully selecting spectral cut off frequencies the required phase information can be separated from the high frequency speckle and DC noise.

By combining the Fourier transform technique with the minimum spanning tree strategy of phase unwrapping a fringe pattern can be solved despite the presence of discontinuities and pixel noise.

The Phase Stepping technique requires either three or four images to extract the phase information, depending on whether a calibrated phase step is used.

As multiple images are used the resolution of the phase information is an order of magnitude higher than that of the Fourier transform technique, images being resolved typically up to 100th of a wavelength.

Due to the requirement of a switching mechanism to generate the phase step, this technique is susceptible to vibration as a beam has to be split into two remote optical paths. A movement of a fraction of a wavelength between these two optical paths will completely mask the phase change to be measured.

In contrast, as only one image is required for the Fourier transform technique a switching mechanism is not required and so neither are remote beams. The apparatus required is therefore not only simpler but is less susceptible to noise.

## 6. User guide for FRAN NT, Version 1.

### 6.1 Introduction

FRAN can separate the phase information from a single finite fringe interferogram using the Fourier Transform method or from three uniformly phase shifted finite or infinite fringe images using the phase stepping method.

The resulting wrapped map of discontinuous tan fringes can be unwrapped using either simple fringe counting or the largely noise immune minimum spanning tree approach. In addition to this, externally produced wrapped phase maps can be unwrapped into a continuous surface.

### 6.2 FILE

**New:-** Sets all the settings to the defaults otherwise the settings will remain the same as those used for the previous image. If the settings have not been reset the previous image will not be displayed as in the Windows 95 version.

**Open:-** Allows a new image to be opened. Does not change settings from previous analysis.

**Save Settings As:-** Allows defaults to be saved with the .CFG extension. If an image is opened with the same filename as the configuration file, the configuration file will be automatically opened. Alternatively the configuration file can be opened with the OPEN command.

**About:-** Author and version information.

**Exit:-** Exits FRAN.

### 6.3 VIEW

**Image Information:-** Gives information on the picture size, the number of bits per pixel and hence the number of colours used, and also a short written description.

**Normal Size:-** Display image with an aspect ratio of one. Is used after the picture has been magnified etc.

**Fit image to window:-** Displays image as large as possible in the current window without distorting its aspect ratio.

**Zoom to cursor defined box:-** A cursor appears to allow the corners of a box to be defined. The display zooms to that area but preserves the aspect ratio. It can be useful

to zoom in on an image to determine the number of pixels per fringe and hence the carrier frequency.

**Zoom In:-** Allows you to select the amount of magnification required from x2 to x10.

**Zoom out:-** Allows you to zoom out from x2 to x10.

**Palette:-** Either a grey scale or a pseudo colour palette can be selected to highlight the features of the current image.

## 6.4 SELECT

**Scan line with undeformed carrier for FFT method:-** A prompt appears with a cursor so that the a scan line can be chosen. The scan line will be used to calculate the carrier frequency so a horizontal raster of pixels with no deformation should be chosen. If an undeformed reference image is used for analysis prior to the analysis of a sample image the carrier frequency calculated from the reference image should also be used on the sample image.

**Analysis of full field:-** The whole image shall be subjected to the same analysis even if it contains large areas of noise or silhouettes.

**Analysis of a cursor defined area:-** Allows a polygon to be defined around the area of interest so that noise can be excluded from the analysis. A minimum of three points are defined using the left mouse key. The sequence is ended using the right mouse key. FRAN then automatically joins up the first and last points using the shortest possible distance.

## 6.5 OPTIONS

**Interferogram pre-processing options:-** If the image contains spike noise it can be passed through filters to try and improve the image. Two such filters are available in FRAN. Firstly, a 3x3 pixel averaging filter, and secondly a 3x3 median filter. In each case FRAN considers all eight of the neighbouring pixels surrounding the pixel in question. The average or median is calculated in relation to these neighbours. The number of iterations of each can be defined. Both filters have the effect of blurring the image to the human eye thus reducing spike noise and the definition of the image. If the image contains any abrupt features such as a shock wave or localised areas of high fringe data the feature may be blurred out. The amount of blurring increases with the number of iterations. If the image shows poor contrast it can be normalised to stretch the grey levels of the image over the entire 256 available rather than just a limited section of this range.

**Phase unwrapping options:-** The approximate size of the large scale discontinuities is asked for. The size is used to define the tile size with typical values being within five to fifty square pixels for a 512 by 512 image. Tile sizes at the extremes or outside

this range may require extensive processing times or even cause the FRAN to roll over. The data within each tile is considered to see whether the tile is valid or not. If the tile contains too much noise it is not included in the unwrap path. The unwrap path can either be a straight scan line as used in simple fringe counting or the path defined according to a minimum spanning tree. The minimum spanning tree strategy allows the data to be successfully unwrapped despite the presence of noise and discontinuities within the fringe data.

**Output file generation options:-** Six output files can be generated by FRAN although at the time of writing problems have been experienced in saving all six of the files in any one analysis. If the analysis fails whilst trying to write the files, subsequent runs will be jeopardised unless the previously failed files of the same name are either renamed or deleted. Five of the six files are in the tagged image file format and are given the .tif extension. These five files are the unwrapped and wrapped phase, given the .UWR and .WRP extensions respectively. The edge detection fringes in the wrapped phase map giving a sobel edge detection of the wrapped phase map with the .EDG extension. The points of low modulation containing the points of low modulation noise in the wrapped phase map with the .MOD extension, and lastly the tile connection tree used to unwrap the phase field can be saved with the .EDG extension. The sixth file is an ASCII file with the .DAT extension which can be read directly into Matlab for post processing. The dat file is a two dimensional matrix of fringe order numbers sampled at a grid spacing specified under pixel sample interval for unwrapped phase. As an alternative to using the dat file the unwrapped tif file can be read into Matlab if the tiffread command has been installed although the tif file will have as many elements as there are pixels within the fringe image. The post processing may therefore be very slow.

## **6.6 ANALYSIS**

**1D FFT method with phase unwrap:-** The file name and path of the image is displayed. All files will be saved under this path. Either the carrier frequency determined from the scan line defined in section 6.4 or an explicitly defined carrier frequency can be used. The explicitly defined carrier frequency may be used when it is necessary to keep the carrier frequency constant between subsequent images. To separate the phase information from the carrier frequency, dc and high frequency noise the phase information has to be isolated. This can either be done automatically or defined explicitly. For automatic isolation min and max are set to 0 and 1 respectively. For explicit definition min and max have to be defined within this range. The FFT data containing the spectral data about the raster scan lines between the rasters 'start scan' and 'end scan' can be saved. Several file types are created.

**3 bucket phase step method with phase unwrap:-** The uniform phase step between the three subsequent images is required in degrees. The path and file name of the first image is required together with the file names of the last two images.

**Phase unwrap externally generated phase map:-** An externally generated wrapped phase map in binary format containing single precision floating point numbers within the range of  $-\pi$  to  $\pi$  can be unwrapped. The image dimensions in pixels is also required. The file must have the .FWR extension.

### **6.7 Procedure for using the FFT method.**

- a) Open the file.
- b) Display the image in a suitable format either by zooming in or changing the palette if required.
- c) If the carrier frequency is to be determined using a scan line position the scan line across an undeformed area of the image.
- d) If a scan line is not to be used either use a predetermined carrier frequency or zoom in on an undeformed area and count the number of pixels per fringe over as many fringes as possible and average.
- e) Define whether the full field or a cursor defined area that excludes noise and reduces processing time is to be used.
- f) If any pre-processing filters are required to improve the image state how many iterations are required. Normalise the intensities if the image is of poor contrast.
- g) State whether simple fringe counting or the minimum spanning tree approach to phase unwrapping is required. Also state the size of the large scale discontinuities so that the tile size can be determined.
- h) State the output files required. It is recommend that only the files definitely required are saved to avoid using too much disk space and to reduce the chances of FRAN rolling over.
- i) Choose the one dimensional FFT method of analysis. State whether a scan line or explicitly defined carrier frequency is to be used. Also state whether the phase information is to be isolated automatically or explicitly defined. Adjust min and max to 0 and 1 for automatic isolation or within this range for explicit definition.

### **6.8 Procedure for using the phase stepping method.**

- a) Open the first of the three phase stepped images.
- b) Display the image in a suitable format either by zooming in or changing the palette if required.
- c) Define whether the full field or a cursor defined area that excludes noise and reduces processing time is to be used.
- d) If any pre-processing filters are required to improve the image state how many iterations are required. Normalise the intensities if the image is of poor contrast.
- e) State whether simple fringe counting or the minimum spanning tree approach to phase unwrapping is required. Also state the size of the large scale discontinuities so that the tile size can be determined.
- f) State the output files required. It is recommend that only the files definitely required are saved to avoid using too much disk space.
- g) State the phase step in degrees, the path and file names of the three uniformly phase stepped images.

## **7. References.**

- [1] C. Quan, T.R. Judge, 'The Processing of the Interference Fringe Data by FFT Method.' Handout for MSc in Optical Engineering.
- [2] P.J. Bryanston-Cross, C. Quan, T.R. Judge, 'Application of the FFT method for the quantitative extraction of information from high resolution interferometric and photoelastic data.' *Optics and Laser Technology*, Vol. **26**, No. **3**, 1994.
- [3] T.R. Judge, 'FRANSYS fringe Analysis System Documentation.' Optical Engineering Laboratory, Dept. of Eng., University of Warwick.
- [4] D.P. Towers, P.J. Bryanston-Cross, C.E. Towers, 'The Automatic Quantitative Analysis of Phase Stepped Interferograms.' Optical Engineering Laboratory, Dept. of Eng., University of Warwick.
- [5] D.P. Towers, T.R. Judge, P.J. Bryanston-Cross, 'Analysis of Holographic Fringe Data Using the Dual Reference Approach.' *Optical Eng.*, Vol. **30**, No. **4**, 1991.